

$$(x_1 = \phi_1 \cos \phi_2, x_2 = \phi_1 \sin \phi_2 \cos \phi_3, \dots, x_{n-1} = \phi_1 \sin \phi_2 \dots \sin \phi_{n-1} \cos \phi_n, x_n = \phi_1 \sin \phi_2 \dots \sin \phi_{n-1} \sin \phi_n)$$

$$\Delta f = \sum_{i=1}^n \frac{\partial}{\partial x_i} \left(\frac{\partial f}{\partial x_i} \right)$$

$$\Delta f = \sum_{i=1}^n \frac{\partial}{\partial x_i} \left(\sum_{j=1}^n \left(\frac{\partial f}{\partial \phi_j} \times \frac{\partial \phi_j}{\partial x_i} \right) \right)$$

$$\Delta f = \sum_{i=1}^n \sum_{j=1}^n \frac{\partial}{\partial x_i} \left(\frac{\partial f}{\partial \phi_j} \times \frac{\partial \phi_j}{\partial x_i} \right)$$

$$\Delta f = \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial}{\partial x_i} \left(\frac{\partial f}{\partial \phi_j} \right) \times \frac{\partial \phi_j}{\partial x_i} + \frac{\partial f}{\partial \phi_j} \times \frac{\partial}{\partial x_i} \left(\frac{\partial \phi_j}{\partial x_i} \right) \right)$$

$$\Delta f = \sum_{i=1}^n \sum_{j=1}^n \left(\left(\sum_{k=1}^n \frac{\partial}{\partial \phi_k} \left(\frac{\partial f}{\partial \phi_j} \right) \times \frac{\partial \phi_k}{\partial x_i} \right) \times \frac{\partial \phi_j}{\partial x_i} + \frac{\partial f}{\partial \phi_j} \times \left(\sum_{k=1}^n \frac{\partial}{\partial \phi_k} \left(\frac{\partial \phi_j}{\partial x_i} \right) \times \frac{\partial \phi_k}{\partial x_i} \right) \right)$$

$$\Delta f = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \left(\frac{\partial}{\partial \phi_k} \left(\frac{\partial f}{\partial \phi_j} \right) \times \frac{\partial \phi_k}{\partial x_i} \times \frac{\partial \phi_j}{\partial x_i} + \frac{\partial f}{\partial \phi_j} \times \frac{\partial}{\partial \phi_k} \left(\frac{\partial \phi_j}{\partial x_i} \right) \times \frac{\partial \phi_k}{\partial x_i} \right)$$

$$\Delta f = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \left(\frac{\partial}{\partial \phi_k} \left(\frac{\partial f}{\partial \phi_j} \right) \times \frac{\partial \phi_k}{\partial x_i} \times \frac{\partial \phi_j}{\partial x_i} \right) + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \left(\frac{\partial f}{\partial \phi_j} \times \frac{\partial}{\partial \phi_k} \left(\frac{\partial \phi_j}{\partial x_i} \right) \times \frac{\partial \phi_k}{\partial x_i} \right) +$$

$$\sum_{j=1}^n \sum_{k=1}^n \left(\sum_{i=1}^n \left(\frac{\partial}{\partial \phi_k} \left(\frac{\partial f}{\partial \phi_j} \right) \times \frac{\partial \phi_k}{\partial x_i} \times \frac{\partial \phi_j}{\partial x_i} \right) \right) + \sum_{j=1}^n \left(\sum_{i=1}^n \sum_{k=1}^n \left(\frac{\partial f}{\partial \phi_j} \times \frac{\partial}{\partial \phi_k} \left(\frac{\partial \phi_j}{\partial x_i} \right) \times \frac{\partial \phi_k}{\partial x_i} \right) \right)$$

$$\sum_{j=1}^n \sum_{k=1}^n \left(\frac{\partial}{\partial \phi_k} \left(\frac{\partial f}{\partial \phi_j} \right) \times \sum_{i=1}^n \left(\frac{\partial \phi_k}{\partial x_i} \times \frac{\partial \phi_j}{\partial x_i} \right) \right) + \sum_{j=1}^n \left(\frac{\partial f}{\partial \phi_j} \times \sum_{i=1}^n \sum_{k=1}^n \left(\frac{\partial}{\partial \phi_k} \left(\frac{\partial \phi_j}{\partial x_i} \right) \times \frac{\partial \phi_k}{\partial x_i} \right) \right)$$

$$\sum_{j=1}^n \sum_{i=1}^n \left(\frac{\partial}{\partial \phi_i} \left(\frac{\partial f}{\partial \phi_j} \right) \times \sum_{k=1}^n \left(\frac{\partial \phi_i}{\partial x_k} \times \frac{\partial \phi_j}{\partial x_k} \right) \right) + \sum_{k=1}^n \left(\frac{\partial f}{\partial \phi_k} \times \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial}{\partial \phi_j} \left(\frac{\partial \phi_k}{\partial x_i} \right) \times \frac{\partial \phi_j}{\partial x_i} \right) \right)$$

$$\Delta f = \sum_{i=1}^n \left(\frac{\partial^2 f}{\partial \phi_i^2} \times \left(\frac{\partial \phi_i}{\partial x_i} \right)^2 \right) + \sum_{k=1}^n \left(\frac{\partial f}{\partial \phi_k} \times \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial}{\partial \phi_j} \left(\frac{\partial \phi_k}{\partial x_i} \right) \times \frac{\partial \phi_j}{\partial x_i} \right) \right)$$

$$\Delta f = \frac{\partial^2 f}{\partial \phi_1^2} + \frac{1}{\phi_1^2} \times \frac{\partial^2 f}{\partial \phi_2^2} + \sum_{i=3}^n \left(\frac{1}{\phi_1^2} \times \left(\prod_{k=2}^{i-1} \frac{1}{\sin^2 \phi_k} \right) \times \frac{\partial^2 f}{\partial \phi_i^2} \right) + \frac{n-1}{\phi_1} \times \frac{\partial f}{\partial \phi_1} + \frac{1}{\phi_1^2} \times \frac{n-2}{\tan \phi_2} \times \frac{\partial f}{\partial \phi_2} + \sum_{i=3}^n \left(\frac{1}{\phi_1^2} \times \frac{n-i}{\tan \phi_i} \times \left(\prod_{k=2}^{i-1} \frac{1}{\sin^2 \phi_k} \right) \times \frac{\partial f}{\partial \phi_i} \right)$$

$$\Delta f = \frac{\partial^2 f}{\partial \phi_1^2} + \frac{n-1}{\phi_1} \times \frac{\partial f}{\partial \phi_1} + \frac{1}{\phi_1^2} \times \left(\frac{\partial^2 f}{\partial \phi_2^2} + \frac{n-2}{\tan \phi_2} \times \frac{\partial f}{\partial \phi_2} \right) + \frac{1}{\phi_1^2} \times \sum_{i=3}^n \left(\left(\prod_{k=2}^{i-1} \frac{1}{\sin^2 \phi_k} \right) \times \left(\frac{\partial^2 f}{\partial \phi_i^2} + \frac{n-i}{\tan \phi_i} \times \frac{\partial f}{\partial \phi_i} \right) \right)$$