$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}$$

$$=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial r}\times\frac{\partial r}{\partial x}+\frac{\partial f}{\partial \theta}\times\frac{\partial \theta}{\partial x}+\frac{\partial f}{\partial \varphi}\times\frac{\partial \varphi}{\partial x}\right)$$
$$+\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial r}\times\frac{\partial r}{\partial y}+\frac{\partial f}{\partial \theta}\times\frac{\partial \theta}{\partial y}+\frac{\partial f}{\partial \varphi}\times\frac{\partial \varphi}{\partial y}\right)$$
$$+\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial r}\times\frac{\partial r}{\partial z}+\frac{\partial f}{\partial \theta}\times\frac{\partial \theta}{\partial z}+\frac{\partial f}{\partial \varphi}\times\frac{\partial \varphi}{\partial z}\right)$$

$$=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial r}\times\frac{x}{\sqrt{x^2+y^2+z^2}}+\frac{\partial f}{\partial \theta}\times\frac{-y}{x^2+y^2}+\frac{\partial f}{\partial \varphi}\times\frac{xz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right)$$
$$+\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial r}\times\frac{y}{\sqrt{x^2+y^2+z^2}}+\frac{\partial f}{\partial \theta}\times\frac{x}{x^2+y^2}+\frac{\partial f}{\partial \varphi}\times\frac{yz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right)$$
$$+\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial r}\times\frac{z}{\sqrt{x^2+y^2+z^2}}+\frac{\partial f}{\partial \varphi}\times\left(\frac{-\sqrt{x^2+y^2}}{x^2+y^2+z^2}\right)\right)$$

$$=\left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right)\times\frac{\partial r}{\partial x}+\frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial r}\right)\times\frac{\partial \theta}{\partial x}+\frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial r}\right)\times\frac{\partial \varphi}{\partial x}\right]\times\frac{x}{\sqrt{x^2+y^2+z^2}}+\frac{\partial f}{\partial r}\times\frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)$$
$$+\left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \theta}\right)\times\frac{\partial r}{\partial x}+\frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right)\times\frac{\partial \theta}{\partial x}+\frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \theta}\right)\times\frac{\partial \varphi}{\partial x}\right]\times\frac{-y}{x^2+y^2}+\frac{\partial f}{\partial \theta}\times\frac{\partial}{\partial x}\left(\frac{-y}{x^2+y^2}\right)$$
$$+\left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \varphi}\right)\times\frac{\partial r}{\partial x}+\frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \varphi}\right)\times\frac{\partial \theta}{\partial x}+\frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right)\times\frac{\partial \varphi}{\partial x}\right]\times\frac{xz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}+\frac{\partial f}{\partial \varphi}\times\frac{\partial}{\partial x}\left(\frac{xz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right)$$
$$+\left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right)\times\frac{\partial r}{\partial y}+\frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial r}\right)\times\frac{\partial \theta}{\partial y}+\frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial r}\right)\times\frac{\partial \varphi}{\partial y}\right]\times\frac{y}{\sqrt{x^2+y^2+z^2}}+\frac{\partial f}{\partial r}\times\frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)$$
$$+\left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \theta}\right)\times\frac{\partial r}{\partial y}+\frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right)\times\frac{\partial \theta}{\partial y}+\frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \theta}\right)\times\frac{\partial \varphi}{\partial y}\right]\times\frac{x}{x^2+y^2}+\frac{\partial f}{\partial \theta}\times\frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)$$
$$+\left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \varphi}\right)\times\frac{\partial r}{\partial y}+\frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \varphi}\right)\times\frac{\partial \theta}{\partial y}+\frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right)\times\frac{\partial \varphi}{\partial y}\right]\times\frac{yz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}+\frac{\partial f}{\partial \varphi}\times\frac{\partial}{\partial y}\left(\frac{yz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right)$$
$$+\left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right)\times\frac{\partial r}{\partial z}+\frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial r}\right)\times\frac{\partial \theta}{\partial z}+\frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial r}\right)\times\frac{\partial \varphi}{\partial z}\right]\times\frac{z}{\sqrt{x^2+y^2+z^2}}+\frac{\partial f}{\partial r}\times\frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$
$$+\left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \varphi}\right)\times\frac{\partial r}{\partial z}+\frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \varphi}\right)\times\frac{\partial \theta}{\partial z}+\frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right)\times\frac{\partial \varphi}{\partial z}\right]\times\left(\frac{-\sqrt{x^2+y^2}}{x^2+y^2+z^2}\right)+\frac{\partial f}{\partial \varphi}\times\frac{\partial}{\partial z}\left(\frac{-\sqrt{x^2+y^2}}{x^2+y^2+z^2}\right)$$

$$= \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) \times \frac{x}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial r}\right) \times \frac{-y}{x^2+y^2} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial r}\right) \times \frac{xz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right] \times \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$+ \frac{\partial f}{\partial r} \times \frac{y^2+z^2}{(x^2+y^2+z^2)^{3/2}}$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{x}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{-y}{x^2+y^2} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{xz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right] \times \frac{-y}{x^2+y^2}$$

$$+ \frac{\partial f}{\partial \theta} \times \frac{2xy}{(x^2+y^2)^2}$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{x}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{-y}{x^2+y^2} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{xz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right] \times \frac{xz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}$$

$$+ \frac{\partial f}{\partial \varphi} \times \frac{z[y^2(y^2+z^2-x^2)-2x^4]}{(x^2+y^2+z^2)^2(x^2+y^2)^{3/2}}$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) \times \frac{y}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial r}\right) \times \frac{x}{x^2+y^2} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial r}\right) \times \frac{yz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right] \times \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$+ \frac{\partial f}{\partial r} \times \frac{x^2+z^2}{(x^2+y^2+z^2)^{3/2}}$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{y}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{x}{x^2+y^2} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{yz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right] \times \frac{x}{x^2+y^2}$$

$$+ \frac{\partial f}{\partial \theta} \times \frac{-2xy}{(x^2+y^2)^2}$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{y}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{x}{x^2+y^2} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{yz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}\right] \times \frac{yz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}$$

$$+ \frac{\partial f}{\partial \varphi} \times \frac{z[x^2(x^2+z^2-y^2)-2y^4]}{(x^2+y^2+z^2)^2(x^2+y^2)^{3/2}}$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) \times \frac{z}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial r}\right) \times \frac{-\sqrt{x^2+y^2}}{x^2+y^2+z^2}\right] \times \frac{z}{\sqrt{x^2+y^2+z^2}} + \frac{\partial f}{\partial r} \times \frac{x^2+y^2}{(x^2+y^2+z^2)^{3/2}}$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{z}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{-\sqrt{x^2+y^2}}{x^2+y^2+z^2}\right] \times \left(\frac{-\sqrt{x^2+y^2}}{x^2+y^2+z^2}\right) + \frac{\partial f}{\partial \varphi} \times \frac{2z\sqrt{x^2+y^2}}{(x^2+y^2+z^2)^2}$$

$$= \frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) + \frac{\partial f}{\partial r} \times \frac{2}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{1}{x^2+y^2+z^2} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{1}{x^2+y^2}$$

$$+ \frac{\partial f}{\partial \varphi} \times \left[\frac{z[y^2(y^2+z^2-x^2) - 2x^4]}{(x^2+y^2+z^2)^2(x^2+y^2)^{3/2}} + \frac{z[x^2(x^2+z^2-y^2) - 2y^4]}{(x^2+y^2+z^2)^2(x^2+y^2)^{3/2}} + \frac{2z\sqrt{x^2+y^2}}{(x^2+y^2+z^2)^2} \right]$$

$$= \frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) + \frac{\partial f}{\partial r} \times \frac{2}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{1}{x^2+y^2+z^2} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{1}{x^2+y^2}$$

$$+ \frac{\partial f}{\partial \varphi} \times \frac{z[(y^2(y^2+z^2-x^2) - 2x^4) + (x^2(x^2+z^2-y^2) - 2y^4) + 2(x^2+y^2)^2]}{(x^2+y^2+z^2)^2(x^2+y^2)^{3/2}}$$

$$= \frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) + \frac{\partial f}{\partial r} \times \frac{2}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{1}{x^2+y^2+z^2} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{1}{x^2+y^2}$$

$$+ \frac{\partial f}{\partial \varphi} \times \frac{z[(x^2+y^2)z^2 - 2[(x^2+y^2)^2 - 2x^2y^2] + (x^2-y^2)^2 + 2(x^2+y^2)^2]}{(x^2+y^2+z^2)^2(x^2+y^2)^{3/2}}$$

$$= \frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) + \frac{\partial f}{\partial r} \times \frac{2}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{1}{x^2+y^2+z^2} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{1}{x^2+y^2}$$

$$+ \frac{\partial f}{\partial \varphi} \times \frac{z[(x^2+y^2)z^2 - 2(x^2+y^2)^2 + [(x^2-y^2)^2 + 4x^2y^2] + 2(x^2+y^2)^2]}{(x^2+y^2+z^2)^2(x^2+y^2)^{3/2}}$$

$$= \frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) + \frac{\partial f}{\partial r} \times \frac{2}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{1}{x^2+y^2+z^2} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{1}{x^2+y^2}$$

$$+ \frac{\partial f}{\partial \varphi} \times \frac{z[(x^2+y^2+z^2)(x^2+y^2)]}{(x^2+y^2+z^2)^2(x^2+y^2)^{3/2}}$$

$$= \frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) + \frac{\partial f}{\partial r} \times \frac{2}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial \varphi}\left(\frac{\partial f}{\partial \varphi}\right) \times \frac{1}{x^2+y^2+z^2} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{1}{x^2+y^2}$$

$$+ \frac{\partial f}{\partial \varphi} \times \frac{z}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}$$

$$= \frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{1}{r^2 tan\varphi}\frac{\partial f}{\partial \varphi} + \frac{1}{r^2 sin^2\varphi}\frac{\partial^2 f}{\partial \theta^2}$$