

$$\begin{aligned}
&= \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial r} \right) + \frac{\partial f}{\partial r} \times \frac{3}{\sqrt{x^2 + y^2 + z^2 + t^2}} + \frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial \theta} \right) \times \frac{1}{x^2 + y^2} + \frac{\partial}{\partial \varphi} \left(\frac{\partial f}{\partial \varphi} \right) \times \frac{1}{x^2 + y^2 + z^2} + \frac{\partial}{\partial \psi} \left(\frac{\partial f}{\partial \psi} \right) \times \frac{1}{x^2 + y^2 + z^2 + t^2} \\
&- \frac{\partial f}{\partial \varphi} \times \left[\frac{z(y^2 z^2 + y^4 - x^2 y^2 - 2x^4)}{(x^2 + y^2 + z^2)^2 (x^2 + y^2)^{3/2}} + \frac{z(x^2 z^2 + x^4 - x^2 y^2 - 2y^4)}{(x^2 + y^2 + z^2)^2 (x^2 + y^2)^{3/2}} + \frac{2z\sqrt{x^2 + y^2}}{(x^2 + y^2 + z^2)^2} \right] \\
&- \frac{\partial f}{\partial \psi} \times \left[\frac{t(z^4 + 2y^2 z^2 - x^2 z^2 + z^2 t^2 + y^4 - x^2 y^2 + y^2 t^2 - 2x^4)}{(x^2 + y^2 + z^2 + t^2)^2 (x^2 + y^2 + z^2)^{3/2}} \right] \\
&- \frac{\partial f}{\partial \psi} \times \left[\frac{t(x^4 + 2x^2 z^2 - y^2 z^2 + z^2 t^2 + z^4 - x^2 y^2 + x^2 t^2 - 2y^4)}{(x^2 + y^2 + z^2 + t^2)^2 (x^2 + y^2 + z^2)^{3/2}} \right] \\
&- \frac{\partial f}{\partial \psi} \times \left[\frac{t(y^4 + 2x^2 y^2 - y^2 z^2 + y^2 t^2 + x^4 - x^2 z^2 + x^2 t^2 - 2z^4)}{(x^2 + y^2 + z^2 + t^2)^2 (x^2 + y^2 + z^2)^{3/2}} \right] \\
&- \frac{\partial f}{\partial \psi} \times \left[\frac{2t\sqrt{x^2 + y^2 + z^2}}{(x^2 + y^2 + z^2 + t^2)^2} \right] \\
&= \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial r} \right) + \frac{\partial f}{\partial r} \times \frac{3}{\sqrt{x^2 + y^2 + z^2 + t^2}} + \frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial \theta} \right) \times \frac{1}{x^2 + y^2} + \frac{\partial}{\partial \varphi} \left(\frac{\partial f}{\partial \varphi} \right) \times \frac{1}{x^2 + y^2 + z^2} + \frac{\partial}{\partial \psi} \left(\frac{\partial f}{\partial \psi} \right) \times \frac{1}{x^2 + y^2 + z^2 + t^2} \\
&- \frac{\partial f}{\partial \varphi} \times \frac{z[(y^4 - x^2 y^2 - 2x^4) + (x^4 - x^2 y^2 - 2y^4) + 2(x^2 + y^2)^2]}{(x^2 + y^2 + z^2)^2 (x^2 + y^2)^{3/2}} \\
&- \frac{\partial f}{\partial \psi} \times \frac{t[(z^4 + z^2 t^2 + y^4 + y^2 t^2 - 2x^4) + (x^4 + z^2 t^2 + z^4 + x^2 t^2 - 2y^4) + (y^4 + y^2 t^2 + x^4 + x^2 t^2 - 2z^4) + 2(x^2 + y^2 + z^2)^2]}{(x^2 + y^2 + z^2 + t^2)^2 (x^2 + y^2 + z^2)^{3/2}} \\
&= \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial r} \right) + \frac{\partial f}{\partial r} \times \frac{3}{\sqrt{x^2 + y^2 + z^2 + t^2}} + \frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial \theta} \right) \times \frac{1}{x^2 + y^2} + \frac{\partial}{\partial \varphi} \left(\frac{\partial f}{\partial \varphi} \right) \times \frac{1}{x^2 + y^2 + z^2} + \frac{\partial}{\partial \psi} \left(\frac{\partial f}{\partial \psi} \right) \times \frac{1}{x^2 + y^2 + z^2 + t^2} \\
&- \frac{\partial f}{\partial \varphi} \times \frac{z[(y^2 z^2 + y^4) + (x^2 z^2 + x^4) + 2x^2 y^2]}{(x^2 + y^2 + z^2)^2 (x^2 + y^2)^{3/2}} - \frac{\partial f}{\partial \psi} \times \frac{2t[(z^4 + z^2 t^2) + (y^4 + y^2 t^2) + (x^4 + x^2 t^2) + 2(x^2 y^2 + x^2 z^2 + y^2 z^2)]}{(x^2 + y^2 + z^2 + t^2)^2 (x^2 + y^2 + z^2)^{3/2}} \\
&= \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial r} \right) + \frac{\partial f}{\partial r} \times \frac{3}{\sqrt{x^2 + y^2 + z^2 + t^2}} + \frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial \theta} \right) \times \frac{1}{x^2 + y^2} + \frac{\partial}{\partial \varphi} \left(\frac{\partial f}{\partial \varphi} \right) \times \frac{1}{x^2 + y^2 + z^2} + \frac{\partial}{\partial \psi} \left(\frac{\partial f}{\partial \psi} \right) \times \frac{1}{x^2 + y^2 + z^2 + t^2} \\
&- \frac{\partial f}{\partial \varphi} \times \frac{z(x^2 + y^2)(x^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)^2 (x^2 + y^2)^{3/2}} - \frac{\partial f}{\partial \psi} \times \frac{2t(x^2 + y^2 + z^2)(x^2 + y^2 + z^2 + t^2)}{(x^2 + y^2 + z^2 + t^2)^2 (x^2 + y^2 + z^2)^{3/2}} \\
&= \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial r} \right) + \frac{\partial f}{\partial r} \times \frac{3}{\sqrt{x^2 + y^2 + z^2 + t^2}} + \frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial \theta} \right) \times \frac{1}{x^2 + y^2} + \frac{\partial}{\partial \varphi} \left(\frac{\partial f}{\partial \varphi} \right) \times \frac{1}{x^2 + y^2 + z^2} + \frac{\partial}{\partial \psi} \left(\frac{\partial f}{\partial \psi} \right) \times \frac{1}{x^2 + y^2 + z^2 + t^2} \\
&- \frac{\partial f}{\partial \varphi} \times \frac{z}{(x^2 + y^2 + z^2)\sqrt{x^2 + y^2}} - \frac{\partial f}{\partial \psi} \times \frac{2t}{(x^2 + y^2 + z^2 + t^2)\sqrt{x^2 + y^2 + z^2}} \\
&= \frac{\partial^2 f}{\partial r^2} + \frac{3}{r} \frac{\partial f}{\partial r} + \frac{1}{r^2 \cos^2 \psi \cos^2 \varphi} \frac{\partial^2 f}{\partial \theta^2} + \frac{1}{r^2 \cos^2 \psi} \frac{\partial^2 f}{\partial \varphi^2} - \frac{\tan \varphi}{r^2 \cos^2 \psi} \frac{\partial f}{\partial \varphi} + \frac{1}{r^2} \frac{\partial^2 f}{\partial \psi^2} - \frac{2 \tan \psi}{r^2} \frac{\partial f}{\partial \psi}
\end{aligned}$$